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Reprint

An Investigation of the Relation Among Some of the Statistics for Upper-Air Pressure, Temperature and Density

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ABSTRACT

From analytical and empirical calculations with upper-air pressure, temperature, density and height, it is concluded that: (a) mean density can be estimated closely from mean pressure and temperature, (b) the standard deviation of density can be estimated from mean pressure and temperature and the standard deviation of temperature, (c) the correlation of temperature at two pressure surfaces is a very good estimate of the correlation of density at these two surfaces, but a similar relation does not exist at two constant height surfaces, (d) a linear relation exists between the standard deviation of height of a pressure surface and the standard deviation of pressure at a height equal to the mean height of the pressure surface, (e) there is also some relationship between the correlation of the heights of two pressure surfaces and the correlation of pressures at the monthly mean heights of the pressure surfaces.

These relations provide some short-cut procedures for estimating the statistics from other available statistics. While only three stations and the four midseason months were used to check the relationships, the results seem to substantiate the conclusions.

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1. Introduction

This study was undertaken to investigate the relationship between various statistics along two vertical scales (pressure and height). Some comparisons are made between different statistics on the same vertical scale, and some are made between the same statistics on the two different scales. Previous work along similar lines has been carried out by Dines (1919), Crossley (1950), Brier,[2] Buell (1954), Stidd (1954), Mook (1958), and McRae (1959).

The data which were used to carry out the investigation are:

a) Stations: Miami, Fla.; Columbia, Mo.; Fairbanks, Alaska.
b) Months: January, April, July, October.
c) Period of Record: 1955–1959.
d) Elements on the pressure scale: height, temperature, virtual temperature, density.
e) Pressure levels: 850, 700, 500, 300, 200, 100 mb.
f) Elements on the height scale: pressure, temperature, virtual temperature, density.
g) Height levels: Mean monthly height of each of the above pressure levels at each station. These heights were computed first and then the values of the meteorological elements at these heights were determined and used in the computations of the various statistics on the height scale.

The data were extracted from the original records on file at the National Weather Records Center, Asheville, N. C.

From these data, the mean and standard deviations were computed for each element-station-month-level using two observations per day. In addition, the monthly correlation coefficients of heights, pressures, temperatures, and densities were computed for all possible pairs of levels on the two scales and the intra-level correlation coefficients between temperature and pressure on the constant height scale. These calculations were carried out on the IBM 705 electronic computer by the Climatic Center's Data Processing Division at Asheville, N. C.

With these statistics, the following comparisons were made:

a) $\bar{\rho}$ vs. $\bar{P}/R\bar{T}$ for both height and pressure scales.
b) σ_ρ vs. $\bar{P}\sigma_T/R\bar{T}^2$ for both height and pressure scales.
c) $r(\rho_1\rho_2)$ vs. $r(T_1T_2)$ for both height and pressure scales.
d) Correlation of heights of pressure surfaces versus correlation of pressures at monthly mean height of these surfaces.
e) σ_H vs. σ_P.

2. Comparisons

a.1) $\bar{\rho}$ vs. $P/R\bar{T}$ at constant pressure level. Implied by the work of Dines (1919), Buell (1954), and from

[1] Both authors now with Headquarters, NASA.

[2] Brier, G. W., 1945: Interrelations of pressure, temperature and density in the upper air for North America. Washington, U. S. Weather Bureau, typescript.

experience, the mean density at a constant pressure surface may be approximated by the substitution of the mean temperature into the equation expressing the gas law, that is:

$$\bar{\rho} \approx P/R\bar{T}, \tag{1}$$

where $\bar{\rho}$ is the mean density and $\bar{T}$ is the mean temperature of the constant pressure surface P, and R is the gas constant for air. This approximation is improved if the mean virtual temperature is used instead of the mean temperature.

For each station-month by pressure levels four values of mean density were computed: 1) the summarized mean density, 2) an estimate of the mean density using the mean virtual temperatures, 3) an estimate of the mean density using the mean ambient temperature, and 4) an estimate of the mean density using an estimate of the mean virtual temperature. The estimate of mean virtual temperature was computed from the mean relative humidity given by Ratner,[3] the mean ambient temperature for the five-year period being investigated, and the Smithsonian Meteorological Table Number 72 (List, 1951).

The difference between the computed mean density and the estimate of the mean density using the mean virtual temperature is equal to or less than 0.0005 kg m^{-3} (0.11 per cent) for Miami, 0.0007 kg m^{-3} (0.06 per cent) for Columbia, and 0.0009 kg m^{-3} (0.07 per cent) for Fairbanks. The latitudinal increase in the difference is due to the increase in the variability of density from the tropical to the arctic areas. This variability affects the validity of the approximation Eq (1). The maximum difference between the mean density and the estimate using the mean ambient temperature is 0.0063 kg m^{-3} (0.62 per cent) for Miami, 0.0058 kg m^{-3} (0.57 per cent) for Columbia, and 0.0029 kg m^{-3} (0.28 per cent) for Fairbanks. The maximum differences all occur at the 850-mb level in July for which the effect of moisture was at a maximum. Above the 850-level the differences fall off rapidly and above 700-mb they are in the same range ($\leq$0.0007 kg m^{-3} or 0.1 per cent) as the difference involving the estimates from the mean virtual temperatures. Because of the much larger differences at the 850- and 700-mb levels, an estimate of the mean virtual temperature was computed from a mean relative humidity and used in Eq (1) to test whether the use of available humidity data would improve the estimate. This estimate produced differences equal to or less than 0.0005 kg m^{-3} (0.05 per cent), indicating the utility of, and requirement, for, at least some humidity correction.

The results imply that mean temperature maps for the 500-mb level and above can be relabeled to give accurate mean density maps. The lower levels would require adjustment for the mean humidity pattern.

[3] Ratner, B., 1957: Upper air climatology of the United States. Technical Paper No. 32, U. S. Weather Bureau, 199 pp.

a.2) $\bar{\rho}$ vs. $\bar{P}/R\bar{T}$ at constant height level. Since both pressure and temperature can vary in the daily observations on a constant height level, one would not expect to be able to estimate well the mean density from the equation,

$$\bar{\rho} \approx \bar{P}/R\bar{T}, \tag{2}$$

where the means indicated by a bar refer to the mean values computed from the data for the constant heights. To evaluate the effect of the joint variations of pressure and temperature, estimates of the mean density were computed using the mean virtual temperature, an estimate of the mean virtual temperature, and the mean temperature together with the mean pressure and compared with the actual mean density for the heights.

The difference between the mean density and the estimate of mean density using the mean virtual temperature was equal to or less than 0.0002 kg m^{-3} (0.01 per cent) for Miami, 0.0006 kg m^{-3} (0.06 per cent) for Columbia, and 0.0009 kg m^{-3} (0.09 per cent) for Fairbanks. The difference between the mean density and the estimates of mean density using the estimate of mean virtual temperature was equal to or less than 0.0005 kg m^{-3} (0.05 per cent) for Miami, 0.0004 kg m^{-3} (0.04 per cent) for Columbia, and 0.0008 kg m^{-3} (0.08 per cent) for Fairbanks. The differences between the mean density and the estimate of mean density using the mean temperature were equal to or less than 0.0063 kg m^{-3} (0.63 per cent) for Miami, 0.0025 kg m^{-3} (0.25 per cent) for Columbia, and 0.0015 kg m^{-3} (0.15 per cent) for Fairbanks. Above the mean height of the 700-mb level, the differences using the mean temperature were equal to or less than 0.0006 kg m^{-3} (0.10 per cent).

The over-all results are very similar to the results on the constant pressure levels. The use of mean pressure and mean virtual temperatures gives a mean density estimate of acceptable accuracy at all heights while the estimate using the mean temperature with the mean pressure is acceptable above the 700-mb level mean height. For the 850- and 700-mb level mean heights, estimated mean virtual temperature should be used in place of the mean ambient temperature to obtain useful results.

b.1) σ_ρ vs. $P\sigma_T/R\bar{T}^2$ at constant pressure level. Using Eq (1) and expressing the departures from the mean of individual observations in differential form, we have:

$$(d\rho)/\bar{\rho} \approx -(dT)/\bar{T}. \tag{3}$$

Squaring both sides and summing over N cases:

$$\sum(d\rho)^2/N\bar{\rho}^2 \approx \sum(dT)^2/N\bar{T}^2 \tag{4}$$

or

$$\sigma_\rho^2/\bar{\rho}^2 \approx \sigma_T^2/\bar{T}^2 \tag{5}$$

and

$$\sigma_\rho \approx \bar{\rho}\sigma_T/\bar{T}. \tag{6}$$

Substituting for its estimate $P/R\bar{T}$ we have:

$$\sigma_\rho \approx P\sigma_T/R\bar{T}^2. \quad (7)$$

The utility of Eq (7) was evaluated using the actual standard deviation of density and two estimates. One estimate is based on the insertion of the statistics for virtual temperature in Eq (7) and the other, the statistics for the ambient air temperature.

The maximum differences between the actual standard deviation and the estimate using virtual temperature data were 0.0008 kg m^{-3} (6 per cent) at the 850-mb level. At the higher levels the differences were less than 0.0003 kg m^{-3} (2 per cent). The maximum difference using ambient temperature data was 0.0013 kg m^{-3} (7 per cent) at the 850-mb level. At higher levels the differences at the 850-mb surface cannot be reduced by a correction for mean humidity which proved useful in the estimation of mean density. However, with only a few exceptions, the differences between the computed standard deviations and estimated made by using ambient temperatures are not significantly different from those differences resulting from using virtual temperature. In general, it is felt that either of the estimates are usable

b.2) σ_ρ *vs.* $\bar{P}\sigma_T/R\bar{T}^2$ *at constant height levels.* Using Eq (2) and expressing the departures from the mean of the individual observations in differential form for a constant height surface, we have:

$$d\rho/\bar{\rho} = dP/\bar{P} - dT/\bar{T}. \quad (8)$$

Squaring both sides and summing over N cases:

$$\sum(d\rho)^2/N\bar{\rho}^2 = \sum(dP)^2/N\bar{P}^2 + \sum(dT)^2/N\bar{T}^2 - 2\sum(dT)(dP)/N\bar{P}\bar{T}. \quad (9)$$

Since

$$\sum(dP)(dT)/N = \sigma_P\sigma_T r(TP), \quad (10)$$

then

$$\sigma_\rho^2/\bar{\rho}^2 = \sigma_P^2/\bar{P}^2 + \sigma_T^2/\bar{T}^2 - 2\sigma P\sigma Tr(PT)/\bar{P}\bar{T}. \quad (11)$$

Taking the square root and substituting $\bar{P}/R\bar{T}$ as an estimate of $\bar{\rho}$, Eq (11) becomes

$$\sigma_\rho = (\bar{P}/R\bar{T})[\sigma_P^2/\bar{P}^2 + \sigma_T/\bar{T}^2 - 2\sigma P\sigma Tr(PT)/\bar{P}\bar{T}]^{\frac{1}{2}}. \quad (12)$$

See Mook (1958).

The maximum difference between the estimated [using Eq (12)] and the computed standard deviation of density was equal to or less than 0.0012 km m^{-3} (15 per cent) for Miami, 0.0004 kg m^{-3} (6 per cent) for Columbia, and 0.0006 kg m^{-3} (3 per cent) for Fairbanks. The fact that $r(TP)$ is not normally available will limit usefulness. The size of the term involving $r(TP)$ is too large to neglect, being of the same order as the other two terms.

c.1) $r(\rho_i\rho_j)$ *vs.* $r(T_iT_j)$ *at constant pressure surfaces.* The following Eq (13) results from the application of Eq (3) to two levels (i and j) and the formation of the product of these equations:

$$d\rho_i d\rho_j/\bar{\rho}_i\bar{\rho}_j \approx dT_i dT_j/\bar{T}_i\bar{T}_j. \quad (13)$$

Summing over N observations and dividing by N we have:

$$\sum d\rho_i d\rho_j/N\bar{\rho}_i\bar{\rho}_j = \sum dT_i dT_j/\ N\bar{T}_i\bar{T}_j. \quad (14)$$

This summarization of the cross-products of the deviations from the mean may be expressed in terms of the correlation coefficient and the standard deviations—see Eq (10)—so that:

$$\sigma_{\rho_i}\sigma_{\rho_j} r(\rho_i\rho_j)/\bar{\rho}_i\bar{\rho}_j = \sigma_{T_i}\sigma_{T_j} r(T_iT_j)/T_iT_j, \quad (15)$$

where $r(\rho_i\rho_j)$ and $r(T_iT_j)$ are the correlation coefficients of the density and temperature, respectively. From Eq (6)

$$\sigma_\rho/\bar{\rho} \approx \sigma_T/\bar{T} \quad (16)$$

so that we can reduce Eq (15) to

$$r(\rho_i\rho_j) \approx r(T_iT_j). \quad (17)$$

Although the result would be expected from physical considerations, the accuracy of the approximation must be evaluated empirically since it is dependent upon the validity of Eq (1) and the joint variability of the various elements. A rather extensive evaluation was conducted with the three stations by computing correlation coefficients of temperature and density for all possible pairs of standard pressure surfaces between 850 and 100 mb.

The correlation coefficient of ambient temperature is as good an estimate of the correlation of dry air density as the correlation coefficient of virtual temperature is an estimate of the correlation of moist air density. In all cases for Miami and Columbia, the error of estimate was 0.02 or less, and for Fairbanks 0.03 or less. For more than 90 per cent of the comparisons, the differences were 0.01 or less. The maximum differences between the correlation coefficients of temperature and their corresponding coefficients for virtual temperature were equal to or less than 0.065 for Miami, 0.058 for Columbia, and 0.010 for Fairbanks. For all stations, more than 50 per cent of the differences were equal to or less than 0.02. The differences between temperature and true density were of the same order.

The differences between temperature and virtual temperature correlations are due to the variations in the moisture content of the air. The differences between temperature and dry air density correlations are due to the nonlinearity of density with respect to the temperature scale, since density is equal to a constant times the reciprocal of temperature. Because of this nonlinearity, the occurrence of extreme values in a sample, which departs widely from the mean, will increase the differences between the two correlation coefficients. The differences between the correlation coefficient of temperature and that of moist air densities are due to both the variation in the moisture content of the air and to the nonlinearity of the density scale.

It appears that for most purposes the inter-pressure surface correlation coefficient of temperatures would be a suitable substitute for the inter-pressure surface correlation coefficient of moist air density.

c.2) $r(\rho_i\rho_j)$ vs. $r(T_iT_j)$ at constant height levels. There is no useful analytical relationship between the inter-height correlation coefficients of temperatures and those of densities, nor between the inter-height correlation of temperatures or densities and the inter-pressure correlation coefficients of temperatures or densities. However, since the statistics were available, a comparison was made to complete the study.

The maximum differences between the inter-height correlation coefficients of temperature and those of virtual temperatures were 0.06 for Miami, 0.05 for Columbia, and 0.01 for Fairbanks. Similar differences occurred between the coefficients of dry and moist air densities. However, the maximum differences between the coefficients of temperatures and those of densities were quite large, being about 0.56 for Miami, 0.95 for Columbia, and 1.10 for Fairbanks. While correlation coefficients for moist and dry air temperatures could be substituted for each other and similarly for moist and dry densities, temperature coefficients are useless as estimates of density correlations for constant height levels.

d) $r(H_iH_j)$ vs. $r(P_iP_j)$. The variation of the actual height of a constant pressure level can be expressed in terms of the variation of pressure of the equivalent constant height level. The equivalent constant height level is defined as the mean monthly height of the constant pressure level for the particular station or location under consideration. The relationship is as follows:

$$dH_i = RT_i dP_i / gP_i, \tag{18}$$

where g is the acceleration of gravity and the others are as defined previously. For small values of dH_i and dT_i around the constant level i, we can approximate T_i, which is the mean temperature of the layer dH, by the T_i observed at the constant pressure level. If we use subscripts i and j to designate two different levels, then the following expression can be developed from (18):

$$\sum_{k=1}^{N} (dH_i dH_j)_k / \Big[\sum_{k=1}^{N} (dH_i)_k{}^2 \sum_{k=1}^{N} (dH_j)_k{}^2\Big]^{\frac{1}{2}} = \sum_{k=1}^{N} (T_iT_j dP_i dP_j)_k / \Big[\sum_{k=1}^{N} (T_i dP_i)_k{}^2 \sum_{k=1}^{N} (T_j dP_j)_k{}^2\Big]^{\frac{1}{2}}. \tag{19}$$

Let $T_i = \bar{T}_i + dT_i$ and $T_j = \bar{T}_j + dT_j$, then Eq (19) reduces to:

$$r(H_iH_j) = \sum_{k=1}^{N} [(\bar{T}_i + dT_i)dP_i(\bar{T}_j + dT_j)dP_j]_k / \Big\{\sum_{k=1}^{N} [(\bar{T}_i + dT_i)^2 dP_i{}^2]_k \sum_{k=1}^{N} [(\bar{T}_j + dT_j)^2 dP_j{}^2]_k\Big\}^{\frac{1}{2}}. \tag{20}$$

Expanding the right side of (20) and dividing both numerator and denominator by $\bar{T}_i\bar{T}_j$,

$$\begin{aligned} r(HH) = \sum_{k=1}^{N} \big[&(dP_i dP_j) + (dT_i dP_i dP_j/\bar{T}_i) \\ &+ (dT_j dP_i dP_j/\bar{T}_j) + (dT_i dT_j dP_i dP_j/\bar{T}_i\bar{T}_j)\big]_k / \\ &\Big\{\sum_{k=1}^{N} [(1 + (2dT_i/\bar{T}_i) + (dT_i{}^2/\bar{T}_i{}^2))dP_i{}^2]_k \\ &\times \sum_{k=1}^{N} [(1 + (2dT_j/\bar{T}_j) + (dT_j{}^2/\bar{T}_j{}^2))dP_j{}^2]k\Big\}^{\frac{1}{2}}. \end{aligned} \tag{21}$$

While the statistics were not available to evaluate each term of this equation, we can estimate the value of the four terms in the numerator of the right-hand side of Eq (21). The first term of the numerator (when divided by the denominator) is approximately $r(P_iP_j)$. The average absolute value of $dT_i/\bar{T}_i$ (or $dT_j/\bar{T}_j$) is about 0.02 or less. The second and third terms are of the order of 0.02 $r(P_iP_j)$ and the last term is insignificant (equal to or less than 0.001 $r(P_iP_j)$. These approximations indicate that $r(H_iH_j) \approx 1.04\ r(P_iP_j)$. Therefore, the absolute value of $r(H_iH_j)$ should be, on the average, larger than the absolute value of $r(P_iP_j)$. For Miami and Columbia the absolute value of $r(H_iH_j)$ was larger than the absolute value of $r(P_iP_j)$ in 85 per cent of the cases, while for Fairbanks it was so in 100 per cent of the cases. The maximum value that $r(H_iH_j)$ exceeded $r(P_iP_j)$ was 0.23 while the largest value of the reverse was 0.08. The largest differences occurred in July when the standard deviations are smallest. This probably reflects the influence of random error in the data, which will have its maximum effect when the standard deviation is smallest. The average amount that $r(H_iH_j)$ exceeded $r(P_iP_j)$ was 0.04. It is felt that for most purposes the interval correlation coefficients of heights would be useful estimates of the interlevel correlation coefficients of pressures.

e) σ_H vs. σ_P. Starting with Eq (18), we can develop a relationship between the standard deviation of height on a constant pressure surface and the standard deviation of pressure on the equivalent constant height level as previously defined. Squaring both sides of Eq (18) and summing over N cases we have

$$\sum dH^2/N = R^2 \sum (TdP)^2/g^2P^2N \tag{22}$$

or

$$\sigma_H{}^2 = R^2 \sum (TdP)^2/g^2P^2N. \tag{23}$$

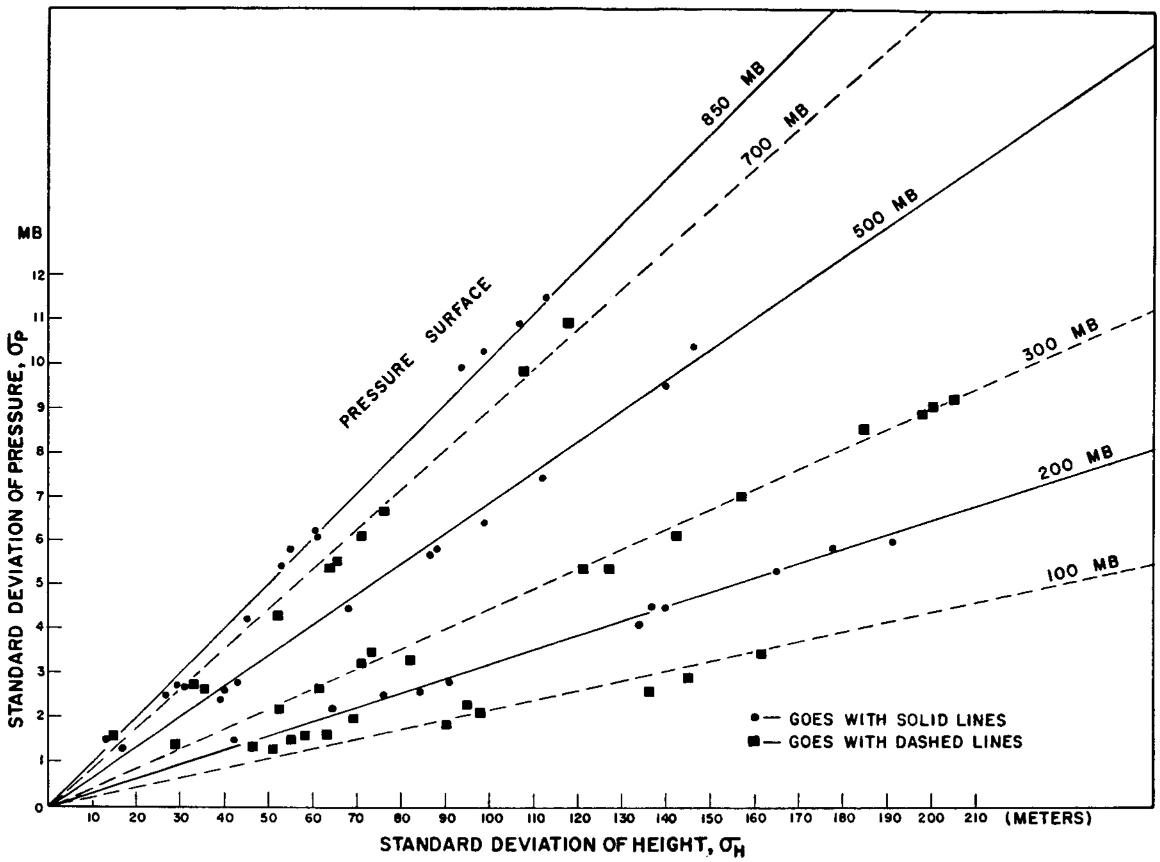

FIG. 1. Relationship between pressure and height standard deviations by pressure levels.

Substituting

$$T=\bar{T}(1+dT/\bar{T}), \tag{24}$$

$$\sigma_H{}^2=R^2\bar{T}^2\sum(1+dT/\bar{T})^2dP^2/g^2P^2N \tag{25}$$

$$\sigma_H{}^2=R^2\bar{T}^2\sum(dp^2+2dTdP^2/\bar{T} +dT^2dP^2/\bar{T}^2)/Ng^2P^2 \tag{26}$$

$$\sigma_H{}^2=(R^2\bar{T}^2/g^2P^2)[\sigma_P{}^2+2\sum dTdP^2/N\bar{T} +\sum dT^2dP^2/N\bar{T}^2]. \tag{27}$$

The second and the third terms in the brackets are negligible compared to the first, and the equation reduces to:

$$\sigma_H\approx(R\bar{T}/gP)\sigma_P \tag{28}$$

or

$$\sigma_p\approx(gP/R\bar{T})\sigma_H. \tag{29}$$

This relation was checked by using Eq (29) to estimate a standard deviation of pressure. The largest error was 0.85 mb at Miami, 0.80 mb at Columbia, and 1.22 mb at Fairbanks. The average difference was 0.25 mb. The 100-mb level had the largest error and in all cases it was an underestimate of the actual standard deviation. No adequate explanation for the large error of estimate at the 100-mb level is available. The authors' opinion is that it may be a reflection of the increasing size of the standard deviation of pressure.

Fig. 1 was prepared by plotting σ_P vs. σ_H for each of the six levels and fitting by eye a regression line to the data for each level. The maximum error from using this graph to estimate σ_P given σ_H is about 0.5 mb and again it occurs at the 100-mb level. Below the 100-mb level the largest error of estimate using the graph is about 0.35 mb.

By substituting the values of the Standard NACA Atmosphere parameters into the factor $gP/R\bar{T}$ from Eq (29) for appropriate levels, we can compute a theoretical slope for the relationship. We can also measure the slope of the regression lines from Fig. 1. The results are as follows:

Level (mb)	Theoretical slope	Empirical slope
850	1/9.8	1/10
700	1/11.2	1/11.4
500	1/14.7	1/15
300	1/22.2	1/21
200	1/32.0	1/30
100	1/63.8	1/40

It is interesting to note the close agreement between the two sets of values of the slopes except for the 100-mb level. Again, we have no explanation for the failure of the 100-mb level relationship not following the pattern of the levels below 100 mb unless it is that the effect of the temperature departure from standard is greater at the 100-mb level than the lower levels.

3. Conclusion

This investigation in itself does not exhaust the relations that may be developed nor the comparisons to be made nor do the results conclusively prove the relationships indicated. However, the use of stations covering a wide range of latitudes and the use of a reasonable period of record give the authors confidence that the the results will most likely hold up through future investigations.

As a consequence of this investigation we can conclude that:

a) $\bar{P}/R\bar{T}$ gives an acceptable estimate of $\bar{\rho}$ for both height and pressure scales.

b) $P\sigma_T/R\bar{T}^2$ gives an acceptable estimate of σ_ρ for the constant pressure surface.

c) $(\bar{P}/R\bar{T})[\sigma_P{}^2/\bar{P}^2+\sigma_T{}^2/\bar{T}^2-2\sigma_P\sigma_T r(TP)/\bar{T}\bar{P}]^{\frac{1}{2}}$ gives an acceptable estimate of σ_ρ for the constant height surface, but the general unavailability of $r(T_iP_i)$ limits its utility.

d) At the levels under consideration, the correlation coefficients of dry air temperatures, of virtual temperatures, of dry air densities, and of moist air densities between two constant pressure surfaces are usable substitutes for one another.

e) The correlation coefficients of dry air temperatures and of virtual air temperatures between two constant height levels are usable substitutes for one another; similarly with the dry air densities and moist air densities. However, temperature correlation coefficients between two constant height levels are not usable estimates of density correlation coefficients between the two levels, and vice versa.

f) While the inter-pressure correlation coefficients of temperatures are crude estimates of the inter-height temperature correlation coefficients, the differences between the inter-pressure and inter-height correlation coefficients of densities are so large as to be useless in estimating one, given the other.

g) The interlevel correlation of heights can be used to estimate the interlevel correlation of pressures; the interlevel correlation of heights being on the average slightly higher than the interlevel correlation of pressure.

h) The standard deviation of heights gives a useful estimate of the standard deviation of pressure, and vice versa.

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